Inclined Road Forces

Gradient Resistance (GR)

The gradient resistance (climbing resistance, inclined road force) depends on the angle of the road inclination and the weight of the car.

$$GR = W \sin\theta = mg \sin\theta$$

where:

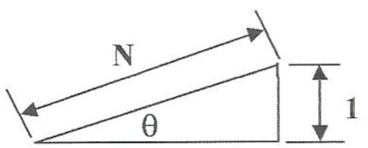
W= the car weight (N) = mg

θ = the angle of road inclination

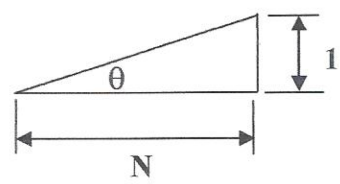
Road inclination (gradient):

The road gradient can be described as 1 in N, this description can be either:

a-



b-



The description (b) is not suitable incase of level road the gradient will be1 in ∞, especially when using the computer. So, the description of the gradient will be as in (a), and the gradient will be ($G = \sin\theta$). The gradient can be written in as a percentage ($G = \sin\theta = S\%$).

* For small angle (θ (radians) $\cong \sin\theta \cong \tan\theta$)
* the road gradient in the highway usually does not exceed 4% and on the local roads it could reach 10%-12%.
* the steepest gradient the vehicle is expected to climb (this may normally be taken as 20%, that is 1 in 5)

The gradient effect on the car:

Up hill:

1- Increase the car motion resistance; $F_G = W \sin\theta$ (against the direction of motion)
2- Increase the load on rear axle and decrease the load on the front one.
3- Decrease the stopping distance when using the brakes (4% is equal to 0.04 g)

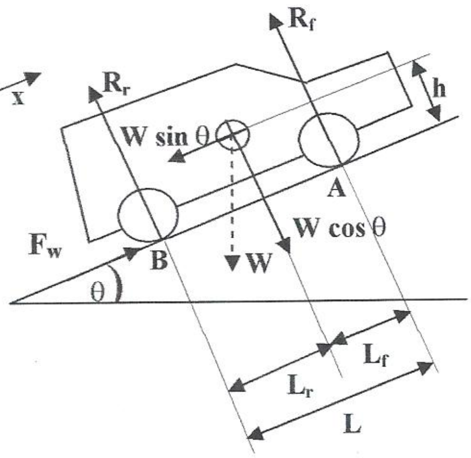
Down hill:

1- Increase the tractive effort; $F_G = W \sin\theta$ (in the direction of motion)
2- Increase the load on front axle and decrease the load on the rear one.
3- Increase the stopping distance when using the brakes (4% is equal to 0.04 g)

Maximum gradient the car can climb (constant velocity)

I-Adhesion

A- Rear wheel drive (neglect the air resistance, and acceleration):



$\Sigma M_A = 0$

$W\cos\theta\ L_f + W\sin\theta\ h\ - R_r L = 0$ .... (A-1)

$\Sigma F_x = 0$

$F_w = W\sin\theta$

But $(F_w)_{max} = \mu R_r = W\sin\theta$

$$R_r = \frac{w\sin\theta}{\mu}$$ .... (A-2)

Substituting Eq. (A-1) into Eq. (A-2)

$$W\cos\theta\, L_f + W\sin\theta\ h - \frac{W\sin\theta}{\mu} L = 0$$ .... (A-3)

Divide Eq. (3) by (W cos θ)

$$L_f + \tan\theta\ h - \tan\theta\ \frac{L}{\mu} = 0$$

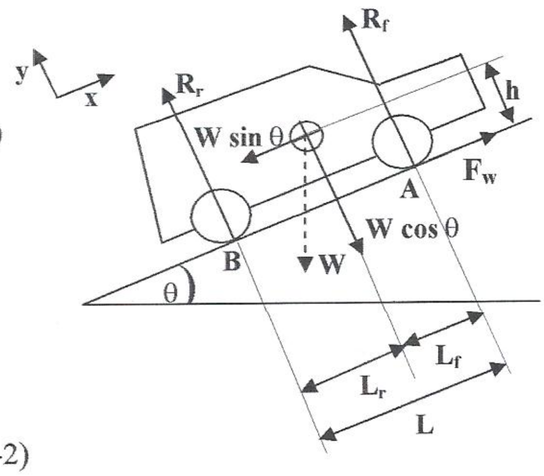
$$L_f = \tan\theta\ \frac{L}{\mu} - \tan\theta\, h = \tan\theta\left(\frac{L - h\mu}{L}\right)$$

$$\tan\theta = \frac{\mu L_f}{L - \mu h}$$ .... (A-4)

Taking rolling resistance ($RR = W f_r$) into consideration:

$$\tan\theta = \frac{\mu L_f - f_r}{L - \mu h}$$ .... (A-5)

B- Front wheel drive (neglect the air resistance, and acceleration):



$\Sigma M_B = 0$

$W \sin\theta \, h - W \cos\theta \, L_r + R_f L = 0$ ....(B-1)

$\Sigma F_x = 0$

$F_w = W \sin\theta$

But $(F_w)_{max} = \mu R_f = W \sin\theta$

$$R_f = \frac{w \sin\theta}{\mu} \quad \text{...........(B-2)}$$

Substituting Eq. (B-1) into Eq. (B-2)

$$W \sin\theta \, h - W \cos\theta \, L_r + \frac{W \sin\theta}{\mu} L = 0 \quad \text{.........(B-3)}$$

Divide Eq. (3) by (W cosθ)

$$\tan\theta \, h - L_r + \tan\theta \frac{L}{\mu} = 0$$

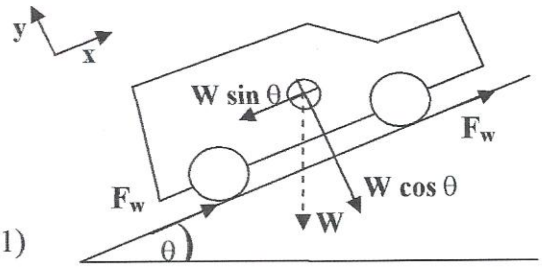
$$L_r = \tan\theta \frac{L}{\mu} + \tan\theta \, h = \tan\theta \left(\frac{L + h\mu}{L}\right)$$

$$\tan\theta = \frac{\mu L_r}{L + \mu h} \quad \text{...........(B-4)}$$

Taking rolling resistance ($RR = W f_r$) into consideration:

$$\tan\theta = \frac{\mu L_r - f_r}{L + \mu h} \quad \text{.............(B-5)}$$

C- Four wheel drive (neglect the air resistance, and acceleration):



$\Sigma F_x = 0$

$F_w = W \sin\theta$

But $(F_w)_{max} = \mu W \cos\theta = W \sin\theta$

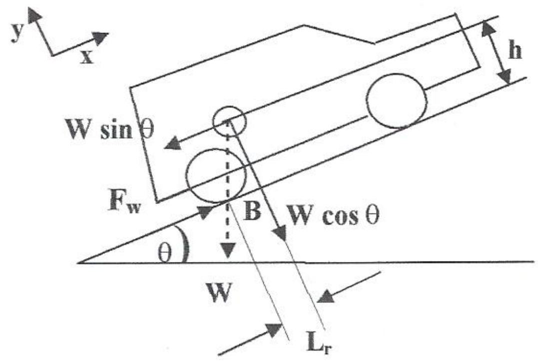
$\tan\theta = \mu$ ....(C-1)

Taking rolling resistance ($RR = W f_r$) into consideration:

$\tan\theta = \mu - f_r$ ....(C-2)

II- Overturn ($R_f = 0$)

a- Constant speed ($F_i = 0$):



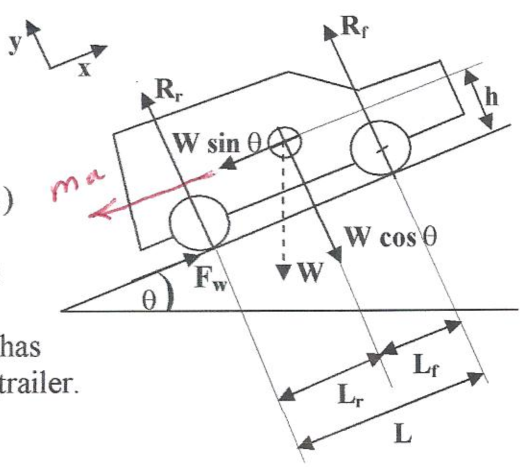
$\Sigma M_B = 0$

$W \sin\theta\, h - W \cos\theta\, L_r = 0$ ....(II-1)

* where $R_f = 0$

$\tan\theta = \frac{L_r}{h} = \frac{L_r/L}{h/L}$

b- Acclerated car ($F_i = M a$):
The worst condition happens at the first gearbox shift, at maximum acceleration. $F_i h = T_w / r_w$



$\Sigma M_B = 0$

$T_{w\,max}\ i_g\ i_f\, h + W \sin\theta\, h - W \cos\theta\, L_r = 0$ r_w

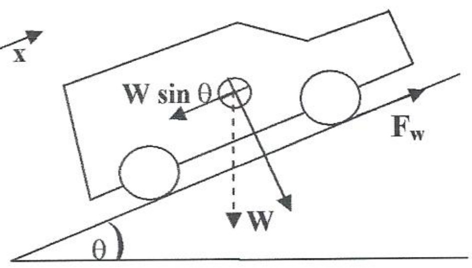
where $R_f = 0$

Overturn condition:

$T_{max}\ i_g\ i_f > W (L_r \cos\theta - h \sin\theta)\ \frac{r_w}{h}$ ....(II-1)

* It is mostly happened to the beach buggy where it has a rear engine (small L_r and L)
* It is also happened to the agricultural tractor where it has a high center of gravity and in case it is coupled to a trailer.

III- Tractive effort



$\Sigma F_x = 0$

$F_w - W \sin\theta = 0$ .... (III-1)

$$\sin\theta = \frac{F_w}{W}$$ .... (III-2)

* where $F_w < \mu W$

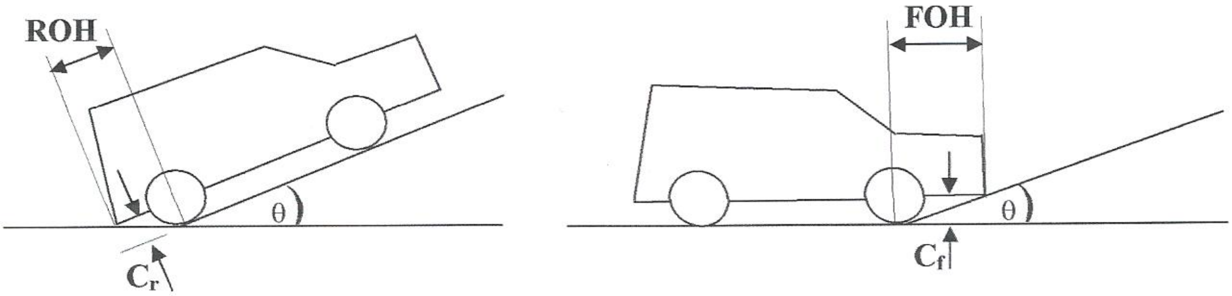
Taking rolling resistance ($RR = W f_r$) into consideration:

$$\sin\theta = \frac{F_w - W f_r}{W}$$ .... (III-3)

IV- Car dimensions

$$\tan\theta = \frac{C_f}{FOH}, \text{ or}$$ .... (IV-1)

$$\tan\theta = \frac{C_r}{ROH}$$ .... (IV-2)



where

FOH = front over hanging
C_f = front under body clearance
ROH = rear over hanging
C_r = rear under body clearance

Example:

A car has the following data:

$m = 1200$ kg	$i_g = (3.2, 2.8, 1.6, 1)$	$R_w = 30$ mm	FOH = 1.3 m
$T_{max} = 180$ N.m	$i_f = 3.8$	$C_f = C_r = 0.35$ m	ROH = 1.0 m
$L = 2.6$ m	$L_f = 1.0$ m	$L_r = 1.6$ m	$h = 0.6$ m

and

$\mu = 0.8$	$f_r = 0.015$

Find the maximum gradient the car can climb at all possibilities.